OFDM De-Noising with RLS Adaptive Filter

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***Abstract*—Orthogonal Frequency Division Multiplexing (OFDM) is designed to improve the spectral efficiency and combat multipath fading by dividing the wide band frequency selective fading channel into many narrow flat sub-channels. The noise in the wireless communication channel may damage the OFDM transmission signal and reduce the signal-to-noise ratio (SNR) at the receiver, which limits the bit error rate (BER) performance and data rate. The adaptive filter has been widely used in random signal processing to deal with random signal and noise. In this paper, we propose a scheme using recursive least square (RLS) adaptive filter to achieve OFDM de-noising in the wireless communication system. RLS filter is an efficient fast convergence data filtering method, which can track the changes of the signal and noise by using exponential weighting and a forgetting factor, and self-adjust the current filter parameters to reach the best filtering performance. Simulation results show that the larger filter length results in better de-noising performance, and the RLS filter can remove the noise of OFDM signal efficiently.**

***Index Terms*—OFDM; adaptive filter; recursive least square (RLS); noise.**

I. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is a high data rate transmission technology in wireless communication systems, which actually comes from multicarrier modulation (MCM) technology and now is considered as key technology in 4G/TD-LTE/LTE-Advanced communication systems [1] [2]. OFDM divides the wide band frequency selective fading channel into many narrow flat sub-channels to combat the effect of multipath reception [3], and it also improves the spectral efficiency and significantly mitigates the intersymbol interference (ISI) [4]. While in the wireless communication system, the interference and noise in the wireless communication channel may damage the OFDM signal and distort the waveform, and OFDM is very sensitive to the interference and noise. The presence of noise will reduce the signal-to-noise ratio (SNR) at the receiver, and consequently, limits the bit error rate (BER) performance and data rate. The interference and noise will change as the mobile stations move, therefore, it is significant to use filter to reduce the interference and noise of OFDM signal in the wireless communication system.

The adaptive filter can deal with random signal, noise and the time-varying signal, and it can track the changes of the signal and noise, therefore, the adaptive filter has have been widely used in random signal processing [5]. Eweda [6] compared the performance of three fundamental adaptive finite impulse response (FIR) filters governed by the recursive least-squares (RLS) algorithm, the least mean square (LMS) algorithm, and the sign algorithm (SA). They found that with appropriate choice of the adaptation parameter, the RLS and LMS algorithms are exponentially convergent, and the convergence rate of the LMS algorithm decreases as the spread of eigenvalues of the input covariance matrix increases, whereas the convergence rate of the RLS algorithm is independent of the eigenvalue spread [6]. Li et al. [7] proposed the RLS channel estimation with superimposed training sequence in OFDM systems while Huang et al. [8] studied the RLS algorithm for active vibration control of smart structures. Xing et al. [5] analyzed the performance of RLS adaptive filter in signal de-noising and found that the high order adaptive filter can remove the noise very well. Ergun et al. [9] used RLS to estimate the channel conditions with training signal for OFDM systems.

In this paper, we propose a adaptive filter scheme to achieve OFDM de-noising in the wireless communication systems since the adaptive filter can track the changes of the signal and noise to achieve optimal filtering, and we prefer the RLS adaptive filter. RLS filter is a fundamental algorithm for adaptive filter, which is an efficient fast convergence data filtering method, and the purpose of RLS filter is to minimize the sum of squares of the error between the desired signal and the output signal to iteratively guarantee each step of the filter parameter can achieve the best value. Therefore, the RLS algorithm is an optimal filter for linear time-invariant systems. As the mobile stations move in the mobile communication, the noise at the mobile stations may change from time to time, so a fixed de-noising filter may fail to perform well. The proposed RLS adaptive filter can track the changes of the signal and noise by using exponential weighting and a forgetting factor, and self-adjust the current filter parameters to reach the best filtering performance. Simulation results show that the larger filter length results in better de-noising performance, and the RLS filter can remove the noise of OFDM signal efficiently.

The remainder of this article is organized as follows. Section II analyzes the derivations of RLS adaptive filter. Section III describes the OFDM communication system using RLS adaptive filter and gives the OFDM de-noising algorithm with RLS filter. Section IV gives the simulation and analysis of OFDM with RLS filter compared to the OFDM without RLS filter under different communication conditions. Finally, Section V warps up the paper with some concluding remarks.

II. THE ANALYSIS OF RLS ADAPTIVE FILTER

Adaptive filter can self-adjust the current filter parameters according to the previous time filter by minimizing a cost function. Fig. 1 is a typical adaptive FIR filter, $\boldsymbol{u}(n)$ is the input signal with noise, $d(n)$ is the reference signal or expected signal and $\boldsymbol{w}(n)$ is the adaptive filter. $y(n)$ is the output signal after filtering from $\boldsymbol{w}(n)$ and $e(n)$ is the error between the desired signal and the output signal. Then we define a cost function $J(n)$ by comparing $d(n)$ and $y(n)$, and we minimize the cost function to get the next filter parameter. The adaptive filter needs little or even no prior knowledge of signal and noise so it is suitable for the mobile changing communication environment. RLS adaptive filter regards the sum of exponential weighting squares of errors as the cost function [5]:

$$J(n) = \sum_{i=0}^{n} \lambda^{n-i} |e(i)|^2 \tag{1}$$

where the weight factor λ typically lies in the range of $0 < \lambda < 1$, and it increases the weight near time n and decreases the weight far away from time n, so we call it forgetting factor. When $\lambda = 1$, the error in every moment is regarded equally, so it does not have forget function. While when $\lambda = 0$, only the current error is considered and the past error is ignored. In order to track the changing system, λ should be neither 0 nor 1, and it is a value nearly equal 1 [10].

The error between the desired signal and the output signal is defined as [11]:

$$e(i) = d(i) - \boldsymbol{w}^H(n)\boldsymbol{u}(i) \tag{2}$$

where $d(i)$ is the expected signal at time i, $\boldsymbol{w}(n)$ is the adaptive filter and $\boldsymbol{u}(i)$ is the input signal at time i. So the complete sum of exponential weighting squares of errors is:

$$J(n) = \sum_{i=0}^{n} \lambda^{n-i} |d(i) - \boldsymbol{w}^H(n)\boldsymbol{u}(i)|^2 \tag{3}$$

Our goal is to minimize this cost function $J(n)$ to update the current filter parameter. First we make the derivative of it as $\frac{\partial J(n)}{\partial \boldsymbol{w}} = 0$, then we have:

$$\boldsymbol{R}(n)\boldsymbol{w}(n) = \boldsymbol{r}(n) \tag{4}$$

Then the filter can be written as:

$$\boldsymbol{w}(n) = \boldsymbol{R}^{-1}(n)\boldsymbol{r}(n) \tag{5}$$

where we can get [7]:

$$\begin{aligned}\boldsymbol{R}(n) &= \sum_{i=0}^{n} \lambda^{n-i}\boldsymbol{u}(i)\boldsymbol{u}^H(i)\\ &= \lambda \sum_{i=0}^{n-1} \lambda^{n-i-1}\boldsymbol{u}(i)\boldsymbol{u}^H(i) + \boldsymbol{u}(n)\boldsymbol{u}^H(n)\\ &= \lambda\boldsymbol{R}(n-1) + \boldsymbol{u}(n)\boldsymbol{u}^H(n)\end{aligned} \tag{6}$$

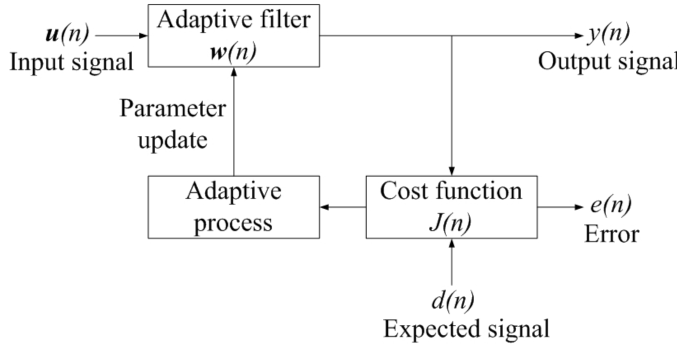

Fig. 1. Adaptive FIR filter.

$$\begin{aligned}\boldsymbol{r}(n) &= \sum_{i=0}^{n} \lambda^{n-i}\boldsymbol{u}(i)d^*(i)\\ &= \lambda \sum_{i=0}^{n-1} \lambda^{n-i-1}\boldsymbol{u}(i)d^*(i) + \boldsymbol{u}(n)d^*(n)\\ &= \lambda\boldsymbol{r}(n-1) + \boldsymbol{u}(n)d^*(n).\end{aligned} \tag{7}$$

We can see that the current $\boldsymbol{R}(n)$ is only decided by the former moment $\boldsymbol{R}(n-1)$ and the current input $\boldsymbol{u}(n)$, and the current $\boldsymbol{r}(n)$ is only decided by the former moment $\boldsymbol{r}(n-1)$, the current input $\boldsymbol{u}(n)$ and the current desired signal $d(n)$. In order to obtain the recursive structure of $\boldsymbol{w}(n)$, we use matrix inversion lemma* to get

$$\begin{aligned}\boldsymbol{R}^{-1}(n) &= \lambda\boldsymbol{R}^{-1}(n-1)\\ &\quad - \frac{\lambda^{-2}\boldsymbol{R}^{-1}(n-1)\boldsymbol{u}(n)\boldsymbol{u}^H(n)\boldsymbol{R}^{-1}(n-1)}{1 + \lambda^{-1}\boldsymbol{u}^H(n)\boldsymbol{R}^{-1}(n-1)\boldsymbol{u}(n)}.\end{aligned} \tag{8}$$

We define $\boldsymbol{P}(n) = \boldsymbol{R}^{-1}(n)$, then we have [11]:

$$\boldsymbol{P}(n) = \lambda^{-1}[\boldsymbol{P}(n-1) - \boldsymbol{k}(n)\boldsymbol{u}^H(n)\boldsymbol{P}(n-1)] \tag{9}$$

where

$$\boldsymbol{k}(n) = \frac{\boldsymbol{P}(n-1)\boldsymbol{u}(n)}{\lambda + \boldsymbol{u}^H(n)\boldsymbol{P}(n-1)\boldsymbol{u}(n)}. \tag{10}$$

and $\boldsymbol{k}(n)$ is called gain vector [12] and it is obvious that:

$$\begin{aligned}\boldsymbol{P}(n)\boldsymbol{u}(n) &= \lambda^{-1}[\boldsymbol{P}(n-1)\boldsymbol{u}(n) - \boldsymbol{k}(n)\boldsymbol{u}^H(n)\boldsymbol{P}(n-1)\boldsymbol{u}(n)]\\ &= \lambda^{-1}\{[\lambda + \boldsymbol{u}^H(n)\boldsymbol{P}(n-1)\boldsymbol{u}(n)]\boldsymbol{k}(n)\\ &\quad - \boldsymbol{k}(n)\boldsymbol{u}^H(n)\boldsymbol{P}(n-1)\boldsymbol{u}(n)\}\\ &= \boldsymbol{k}(n).\end{aligned} \tag{11}$$

Then the filter can be written as:

$$\begin{aligned}\boldsymbol{w}(n) &= \boldsymbol{R}^{-1}(n)\boldsymbol{r}(n) = \boldsymbol{P}(n)\boldsymbol{r}(n)\\ &= \lambda^{-1}[\boldsymbol{P}(n-1) - \boldsymbol{k}(n)\boldsymbol{u}^H(n)\boldsymbol{P}(n-1)]\\ &\quad \cdot [\lambda\boldsymbol{r}(n-1) + \boldsymbol{u}(i)d^*(i)]\\ &= \boldsymbol{P}(n-1)\boldsymbol{r}(n-1) + \lambda^{-1}d^*(i)[\boldsymbol{P}(n-1)\boldsymbol{u}(n)\\ &\quad - \boldsymbol{k}(n)\boldsymbol{u}^H(n)\boldsymbol{P}(n-1)\boldsymbol{u}(n)]\\ &\quad - \boldsymbol{k}(n)\boldsymbol{u}^H(n)\boldsymbol{P}(n-1)\boldsymbol{r}(n-1)\end{aligned} \tag{12}$$

*Such form matrix $(\boldsymbol{A} + \boldsymbol{BCD})$ has the inverse matrix: $(\boldsymbol{A} + \boldsymbol{BCD})^{-1} = \boldsymbol{A}^{-1} - \boldsymbol{A}^{-1}\boldsymbol{B}(\boldsymbol{D}\boldsymbol{A}^{-1}\boldsymbol{B} + \boldsymbol{C}^{-1})^{-1}\boldsymbol{D}\boldsymbol{A}^{-1}$

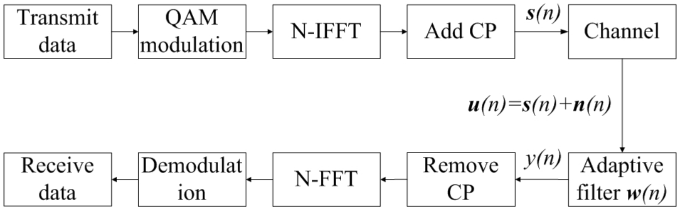

Fig. 2. OFDM communication link with RLS filter.

Therefore, the updated filter is:

$$\begin{aligned}\boldsymbol{w}(n) &= \boldsymbol{w}(n-1) + d^*(n)\boldsymbol{k}(n) - \boldsymbol{k}(n)\boldsymbol{u}^H(n)\boldsymbol{w}(n-1)\\ &= \boldsymbol{w}(n-1) + \boldsymbol{k}(n)\epsilon^*(n) \qquad (13)\end{aligned}$$

where

$$\begin{aligned}\epsilon(n) &= d(n) - \boldsymbol{u}^T(n)\boldsymbol{w}^*(n-1)\\ &= d(n) - \boldsymbol{w}^H(n-1)\boldsymbol{u}(n) \qquad (14)\end{aligned}$$

is the priori error [13].

III. OFDM COMMUNICATION SYSTEM USING RLS ADAPTIVE FILTER

Basically, there are three types of OFDM-based block transmission schemes: cyclic prefix OFDM (CP-OFDM), zero padding OFDM (ZP-OFDM), and time domain synchronous OFDM (TDS-OFDM) [1]. The main idea of OFDM is to divide the wide band frequency selective fading channel into many narrow flat sub-channels, and a typical OFDM symbol is expressed as:

$$s(t) = \sum_{i=0}^{N-1} d_i rect(t - t_s - \frac{T}{2}) e^{j2\pi f_i (t-t_s)}, t_s \leq t \leq t_s + T \qquad (15)$$

where N is the number of subcarriers, d_i denotes the complex data symbol. T is the symbol duration and f_i is the subcarrier frequency. $rect(t)$ is the rectangle function. From the formula we can see that the OFDM is actually the inverse discrete Fourier transform (IDFT) of the data symbol d_i and OFDM modulation can be easily achieved by inverse fast Fourier transform (IFFT) performance. So the demodulation of OFDM can be performed by using FFT.

A complete OFDM communication link with RLS filter is shown in Fig. 2. The transmit data is discrete binary stream, and modulation is to make the signal wave suitable for transmission in the wireless channel. LTE systems use quadrature amplitude modulation (QAM) and we use QAM with different modulation orders in our scheme. IFFT is performed on the modulated signal to make it into OFDM symbol where N is the IFFT/FFT size. Cyclic prefix (CP) is added to avoid intersymbol interference caused by the delay spread of wireless channels [14]. After adding the CP, $\boldsymbol{s}(n)$ is a complete OFDM symbol. The channel is the additive white gaussian noise (AWGN) wireless channel or the multipath wireless channel. We only consider the noise and multipath effect of the channel on the signal, so after passing through the channel, the signal $\boldsymbol{u}(n)$ contains the original signal and the noise $\boldsymbol{n}(n)$, and then we input the receive OFDM signal into the RLS adaptive filter for de-noising.

First we determine the filter length N and initialize the filter $\boldsymbol{w}(0) = \boldsymbol{0}$. The filter with larger filter length may have a better performance, also it becomes more complex and needs more computation, therefore, we balance the performance and the complexity of the filter. Then we initialize $\boldsymbol{P}(0)$, and according to the deduction before we have [13]:

$$\boldsymbol{P}(0) = \boldsymbol{R}^{-1}(0) = \left[\sum_{i=-n}^{0} \lambda^{-i}\boldsymbol{u}(i)\boldsymbol{u}^H(i)\right]^{-1} \qquad (16)$$

$$\boldsymbol{R}(n) = \sum_{i=1}^{n} \lambda^{n-i}\boldsymbol{u}(i)\boldsymbol{u}^H(i) + \boldsymbol{R}(0) \qquad (17)$$

where we want $\boldsymbol{R}(0)$ has little effect in Eq. (17) because of the forgetting effect of λ. Hence, we define $\boldsymbol{R}(0)$ as:

$$\boldsymbol{R}(0) = \delta\boldsymbol{I} \qquad (18)$$

The $\boldsymbol{P}(0)$ is initialized as:

$$\boldsymbol{P}(0) = \delta^{-1}\boldsymbol{I} \qquad (19)$$

where $\boldsymbol{I}$ is a unit matrix and δ is a very small positive value. Usually, δ is typically equivalent to 10^{-4}.

After initialization of the filter parameters, we input received OFDM signal $\boldsymbol{u}(n)$ for adaptive filtering. For the RLS adaptive filtering process, it can be shown as algorithm 1. After adaptive filtering process, the receiver is the inverse process of the transmitter.

Algorithm 1 OFDM de-noising using RLS filter

Initialization: Ensure filter length N, $\boldsymbol{w}(0) = \boldsymbol{0}$, $\boldsymbol{P}(0) = \delta^{-1}\boldsymbol{I}$, where δ is a very small positive value
Input received OFDM signal $\boldsymbol{u}(n)$
Update: n=1,2,3...
Calculate error $\epsilon(n)$: $\epsilon(n) = d(n) - \boldsymbol{w}^H(n-1)\boldsymbol{u}(n)$
Calculate gain vector $\boldsymbol{k}(n)$: $\boldsymbol{k}(n) = \frac{\boldsymbol{P}(n-1)\boldsymbol{u}(n)}{\lambda + \boldsymbol{u}^H(n)\boldsymbol{P}(n-1)\boldsymbol{u}(n)}$
Calculate $\boldsymbol{P}(n)$: $\boldsymbol{P}(n) = \lambda^{-1}[\boldsymbol{P}(n-1) - \boldsymbol{k}(n)\boldsymbol{u}^H(n)\boldsymbol{P}(n-1)]$
Update filter $\boldsymbol{w}(n)$: $\boldsymbol{w}(n) = \boldsymbol{w}(n-1) + \boldsymbol{k}(n)\epsilon^*(n)$
Output OFDM signal $y(n)$: $y(n) = \boldsymbol{w}^H(n)\boldsymbol{u}(n)$

IV. SIMULATION RESULTS AND ANALYSIS

In this paper, the OFDM simulation parameters are shown in table I. We use CP-OFDM in which the CP length can be changed, and the QAM modulation order can be 4, 16 and 64. The adaptive filter length also can be changed, and when the filter length is greater than 128, the filtering process will occupy a large amount of computing resources which can not be realized in the reality. First, Fig. 3 compares the performance of OFDM signal without RLS filter with the performance of OFDM signal using RLS filter under different filter length N. From which we can see the the RLS filter

TABLE I
PARAMETERS OF OFDM LINK.

Parameter	OFDM communication setting
Number of carriers	10000
IFFT/FFT size	10240
Modulation	4|16|64 QAM
Bandwidth	10 MHz
Carrier spacing	1 KHz
Symbol duration	1/1 KHz=1ms
CP size	(1/4|1/8|1/16|1/32)*4096
Channel	AWGN channel or multipath channel+AWGN
Adaptive filter length	4|16|32|64|128

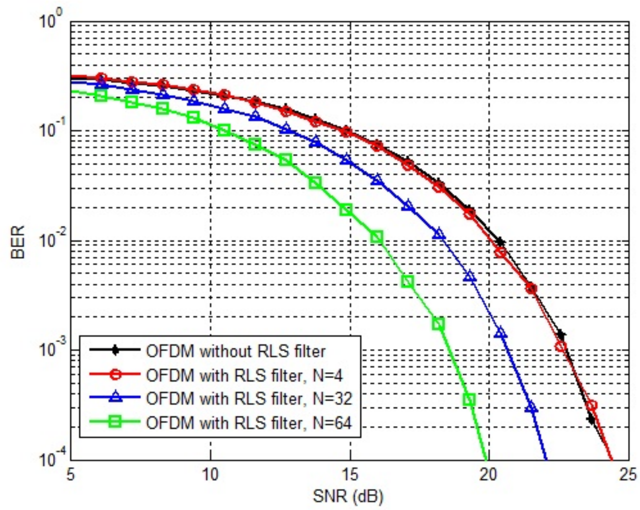

Fig. 3. Comparison of the performance of OFDM signal without RLS filter with performance of OFDM signal using RLS filter under different filter length N.

has seldom de-nosing effect when $N = 4$. While as the filter length increases, the RLS adaptive filter can remove the noise very well and improve the BER performance. The larger filter length results in better de-noising performance, and at the same time the larger filter length needs more computation, therefore, we have to balance the performance and the complexity of the filter.

Fig. 4(a) compares the performance of OFDM signal without RLS filter with the performance of OFDM signal with RLS filter in different QAM modulation orders. And the proposed OFDM de-noising scheme performs well in different QAM modulation orders. Fig. 4(b) draws the complex signal constellation of QAM16. The signal becomes more concentrated to the original signal after passing though the RLS filter, which means the noise of the signal has been removed a lot after RLS filtering. Also, the de-noising performance of the RLS filter in multipath transmission channel has been considered in Fig. 4(c) and (d). Fig. 4(c) shows the comparison of multipath transmission with RLS filter and without RLS filter, and Fig. 4(d) draws the corresponding complex signal constellation. From Fig. 4(c) and Fig. 4(d) we can see the RLS can effectively remove the noise of the transmission signal in multipath channel and improve the BER performance.

V. CONCLUSION

This paper proposed a scheme using RLS adaptive filter to achieve OFDM de-noising in the wireless communication system. The RLS adaptive filter can track the changes of the signal and noise and self-adjust the current filter parameters to reach the best filtering performance. The proposed OFDM de-noising scheme is easy to be implemented and realized. The larger filter length results in better de-noising performance at the cost of large computation and we have to balance the performance and the complexity of the filter. From the simulation results we can see that the RLS adaptive filter can remove the noise of transmission signal very well and improve the BER performance in both different QAM modulation orders and multipath channel.

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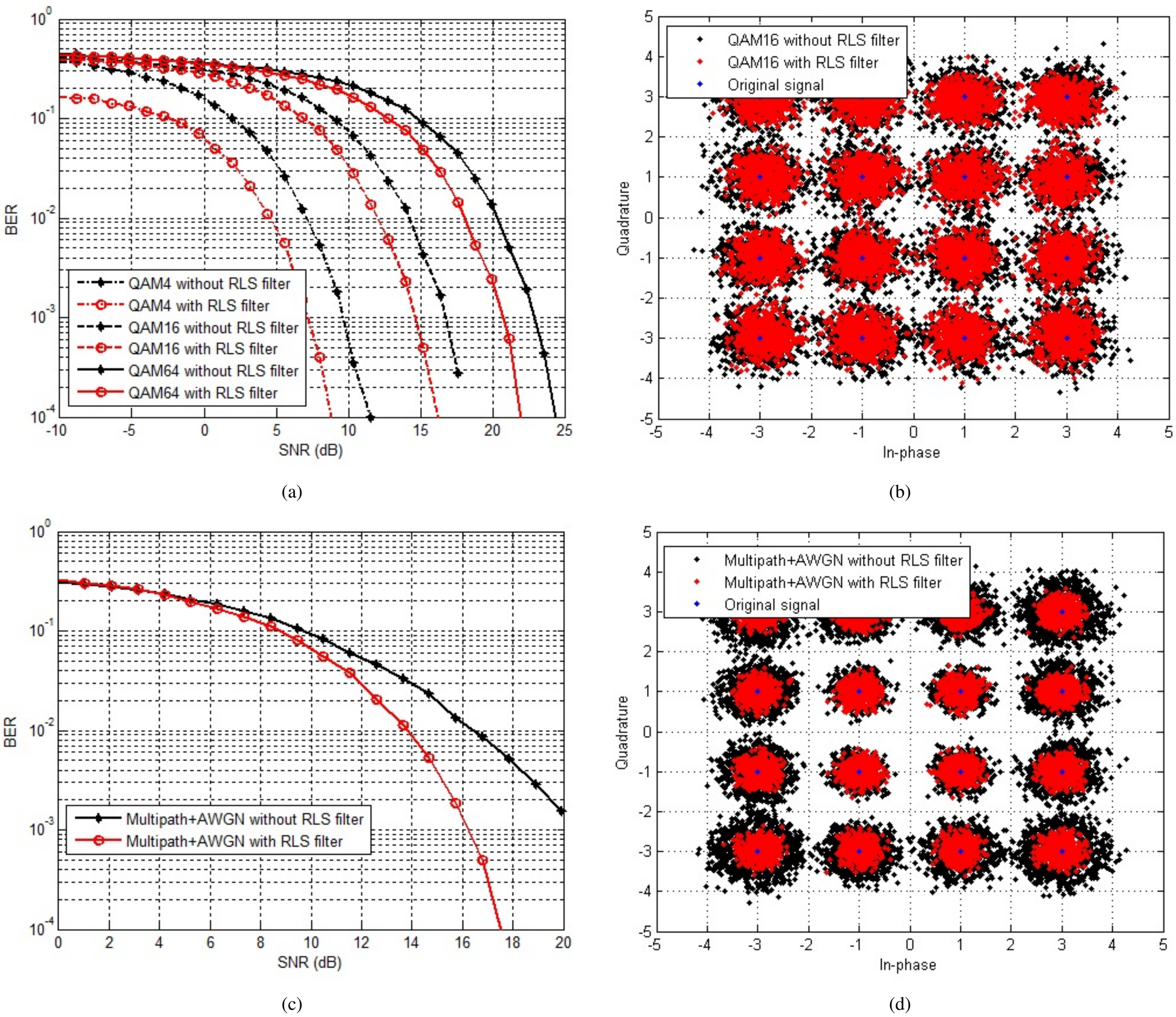

Fig. 4. (a) Comparison of the performance of OFDM signal without RLS filter and the performance of OFDM signal with RLS filter in different QAM modulation orders (b) Complex signal constellation of QAM16 (c) Comparison of multipath transmission with RLS filter and without RLS filter (d) Complex signal constellation of multipath transmission.

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